A New Post-Viterbi Processor Based on Soft-Reliability Information

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Abstract—*This paper proposes a soft-reliability information-based post-Viterbi processor for reducing miss-correction of an error correlation filter-based post-Viterbi processor. The essential difference between the soft-reliability information-based and the error correlation filter-based post-Viterbi processors, is how to locate the most probable error starting position. The new scheme determines an error starting position based on a soft-reliability estimate, while the conventional scheme chooses an error starting position based on likelihood value. Among all likely error starting positions for prescribed error events, the new scheme attempts to correct error-type corresponding to the position only if there exists a position where the soft-reliability estimate is negative, while the conventional scheme performs error correction based on error-type and its error starting position of an error event associated with the maximum likelihood value. A correction made by the conventional scheme may result in miss-correction because the scheme does not have any criterion for judgment whether an estimated error starting position is correct. In case, error correction is only performed when a position with negative soft-reliability estimate exists, the probability of miss-correction of the new scheme is less than the one of the conventional scheme.*

Index Terms — Error detection code, dominant error event, matched-filtering type post-Viterbi processor, **cyclic redundancy check code.**

I. INTRODUCTION

The demand for high-density digital data storage systems has been growing steadily. Although technological innovations in the design of recording media and heads are key to achieving high density recording systems, the role of sophisticated coding and signal processing techniques for data recovery is increasingly becoming crucial in supporting and augmenting these advancements.

To further improve the performance under aggressive recording conditions, currently, there is a great deal of activity exploring turbo coding and soft iterative decoding for future storage products [1]-[3]. Turbo codes and low-density parity-check (LDPC) codes are promising soft error correction codes (ECC) with potential to approach channel capacity. Unfortunately, these schemes have so far not found their way to commercial recording systems as they are complex to implement and show a long latency.

In recent years, there has been a growing interest in error detection codes with error correction properties [4]-[12]. Unlike conventional read channels, where ECC is expected to correct all the errors at the output of the constrained decoder, dominant error events are corrected by applying a low redundancy error detection code. This error detection code is an inner ECC that can correct dominant error events at the output of the channel detector by using only a few parity bits. In this way, the correction capacity loss of the outer ECC is significantly reduced and the error propagation of the modulation decoder is also minimized. The approach using error detection codes, referred to as post-Viterbi processor (in other words, maximum likelihood (ML) post-processor), has found wide acceptance since the performance-complexity trade-off offered is very attractive and affordable. The above approach has been widely studied for magnetic recording channels [5][7]-[12], and for optical recording systems [4][6].

In error correlation filter-based post-Viterbi processor based on an error detection code [4]-[12], when the syndrome is non-zero, the error detection code generates a set of error starting positions of each dominant error event. Each matched filter computes the likelihood (that is, correlation) values over the set of starting positions, and it outputs an error starting position that corresponds to the maximum correlation value. Finally, based on the error-type and its error starting position of the error event that corresponds to the maximum among the outputs of the matched filters, the scheme performs the error correction.

The error correlation filter-based post-Viterbi processor performs error correction based on an estimated error starting position and its error-type whenever the syndrome of a codeword is non-zero. However, the correction may lead to miss-correction because the scheme does not supply any criterion for judgment whether the estimated error-location is precise. The miss-correction is the main source of detrimental factors to the performance improvement. This paper introduces a new technique, the soft-reliability information-based post-Viterbi processor for reducing miss-correction of error correlation filter-based post-Viterbi processor based on error detection code. The new scheme attempts to correct error-type corresponding to the position only if there exists a position where the soft-reliability estimate is negative. The performance has been evaluated for the magnetic recording channel. With only a few alterations, the technique can be applied to optical storage systems.

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The paper is organized as follows. Section II introduces the new technique. In Section III, simulation results are given, and finally, conclusions are given in Section IV.

II. A NEW POST-VITERBI PROCESSOR

This section firstly overviews the error correlation filter-based post-Viterbi processor. Secondly, the soft-reliability information-based post-Viterbi processor will be described. Finally, a worked example of error correction of the new scheme will be shown.

A. Error Correlation Filter-based Post-Viterbi Processor

The performance of a partial response maximum likelihood (PRML) system can be improved by employing an error correlation filter-based post-Viterbi processor based on an error detection code [4]-[12] that can correct a dominant error event at the output of the channel detector (in other words, ML detector). In error correlation filter-based post-Viterbi processors based on error detection code, an error detection decoder computes a syndrome to check for the presence of errors in the estimated codeword, which is found at the output of the channel detector. When the syndrome is non-zero, the scheme is activated. The scheme first calculates the error signal, which is the difference between an output signal of the equalizer and the signal generated by the convolution of the output of the ML detector with the channel target response. The error signal is then passed to a bank of matched filters corresponding to the dominant error events. Here, the matched filter for a given error event is the time-reversed version of the convolution between the error event and the channel target response. Each matched filter computes and outputs the likelihood values over the error starting positions for each dominant error event, where error starting positions are all positions within a codeword [5]-[12] or more probable positions [4]. The outputs (likelihood values) of the matched filters are normalized by subtracting a set of offset values associated with the corresponding error events. The maximum normalized output is used to determine the error-location of the corresponding error event, for each matched-filter. The resulting normalized maximum outputs of all the filters are compared and the largest one is selected. Finally, based on the error-type and its error starting position of the error event corresponding to the largest normalized output, the error event is corrected. It is observed that there has been miss-correction of an error correlation filter-based post-Viterbi processor due to correlated noise and residual inter-symbol interference [6][7]. Now suppose that the actual error position is judged as incorrect one due to channel impairments. Then, the conventional scheme corrects the error-type corresponding to a misjudged error-position and consequently, this results in a miss-correction so that the resulting number of bit errors is increased. The main reason is that the conventional scheme does not supply any criterion for judgment whether an estimated error-location is accurate.

B. Soft-reliability Information-based Post-Viterbi Processor

We describe a sub-optimum post-Viterbi error correction scheme based on soft-reliability information. We assume that a codeword generated by an error detection encoder is transmitted over a partial response channel and corrupted by additive white Gaussian noise (AWGN).

The *k*-th input signal of the ML detector, r_k is expressed as

$$r_k = \sum_{i=0}^{l_h-1} h_i \cdot b_{k-i} + n_k = s_k + n_k , \qquad (1)$$

where b_k is a bipolar coded bit at time *k*, h_k is a channel target response of length l_h , n_k is AWGN, and s_k is the signal sample which is the convolution of the bipolar coded bit b_k and the channel target response h_k . As we know, the ML detector selects a coded bit sequence $\hat{b}_k$ ($k = 0, \cdots, N-1$) that minimizes the Euclidean metric

$$\sum_{k=0}^{N-1} \left(r_k - \hat{s}_k \right)^2 , \qquad (2)$$

where $\hat{s}_k$ is a signal sequence among the convolutions of the output bit of ML detector $\hat{b}_k$ and the channel target response h_k , i.e.,

$$\hat{s}_k = \sum_{i=0}^{l_h-1} h_i \cdot \hat{b}_{k-i} \ . \qquad (3)$$

We try to design a decoding scheme that corrects a prescribed set of error events occurred at the output of the ML detector that dominates the other error events. Let us assume that one of the prescribed error events $\left\{ \mathbf{e}^{(i)},\ i \in \{1, \cdots, E\} \right\}$ occurs in a codeword, where *E* is the number of prescribed error events. Then, L_i likely error starting positions $\left\{ p_j^{(i)},\ i \in \{1, \cdots, E\} \text{ and } j = 1, \cdots, L_i \right\}$ for $\left\{ \mathbf{e}^{(i)},\ i \in \{1, \cdots, E\} \right\}$ from a syndrome computed by error detection code are given, where L_i is the number of likely error starting positions for a prescribed *i*-th error event. If the length of the error event $\mathbf{e}^{(i)}$ is $l^{(i)}$, and its starting position is $p_m^{(i)}$ among $\left\{ p_j^{(i)},\ j = 1, \cdots, L_i \right\}$, where $1 \le m \le L_i$, then the error event $\mathbf{e}^{(i)}$ is given by

$$\begin{aligned} \mathbf{e}^{(i)} &= \left[e_0^{(i)}, e_1^{(i)}, \cdots, e_{l^{(i)}-1}^{(i)} \right] \\ &= \left[b_{p_m^{(i)}}, b_{p_m^{(i)}+1}, \cdots, b_{p_m^{(i)}+l^{(i)}-1} \right] - \left[\hat{b}_{p_m^{(i)}}, \hat{b}_{p_m^{(i)}+1}, \cdots, \hat{b}_{p_m^{(i)}+l^{(i)}-1} \right] \qquad (4) \\ &= \mathbf{b}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1} - \hat{\mathbf{b}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1} \end{aligned}$$

and the error signal vector $\mathbf{s}^{e^{(i)}}$, which is the convolution of the error event $\mathbf{e}^{(i)}$ and the channel target response $\mathbf{h}_0^{l_h-1}$, is expressed as

$$\begin{aligned}\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} &= \left[\mathbf{b}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1} - \hat{\mathbf{b}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}\right] * \mathbf{h}_0^{l_h-1} \\ &= \mathbf{s}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} - \hat{\mathbf{s}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\end{aligned} . \quad (5)$$

Here, each error signal sample $s_k^{e^{(i)}}$ is

$$\begin{aligned} s_k^{e^{(i)}} &= \sum_{i=0}^{l_h-1}\left(b_{k-i} - \hat{b}_{k-i}\right)\cdot h_i \\ &= \sum_{i=0}^{l_h-1} e_{k-p_m^{(i)}-i}^{(i)}\cdot h_i \;\; \text{for } \; p_m^{(i)} \le k \le p_m^{(i)} + l^{(i)} + l_h - 2 \end{aligned}, \quad (6)$$

where $e_j^{(i)} = 0$ if $j < 0$ or $j > l^{(i)} - 1$. Let $\hat{s}'_k$ be the convolution of the flipped ML detector output-bit $\hat{b}'_k$ according to an error event $\mathbf{e}^{(i)}$ and the channel $\mathbf{h}_0^{l_h-1}$, i.e., $\hat{s}'_k = \sum_{i=0}^{l_h-1}\hat{b}'_{k-i}\cdot h_i$. For example, if the type of an error event is known as [2,-2], and one of its likely error starting positions is m, then $\hat{\mathbf{b}}'^{m+1}_m = -\hat{\mathbf{b}}_m^{m+1}$.

It is more likely for one of the prescribed error events $\mathbf{e}^{(i)}$ to start at $p_m^{(i)}$ among all likely positions for prescribed error events through the ratio of the posteriori probabilities, i.e., soft-reliability information, if $\hat{\mathrm{P}} = p_m^{(i)}$, where

$$\hat{\mathrm{P}} = \underset{\text{all } p_{j\in\{1,\dots,L_i\}}^{i\in\{1,\dots,E\}}}{\arg\min}\left(\frac{P\left(\hat{s}_{p_j^{(i)}},\cdots,\hat{s}_{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\, r_{p_j^{(i)}},\cdots,r_{p_j^{(i)}+l^{(i)}+l_h-2}\right)}{P\left(\hat{s}'_{p_j^{(i)}},\cdots,\hat{s}'_{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\, r_{p_j^{(i)}},\cdots,r_{p_j^{(i)}+l^{(i)}+l_h-2}\right)} < 1\right). \quad (7)$$

$P\left(\hat{s}_{p_j^{(i)}},\cdots,\hat{s}_{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\, r_{p_j^{(i)}},\cdots,r_{p_j^{(i)}+l^{(i)}+l_h-2}\right) = P\left(\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)$ can be written by using Bayes' rule as

$$P\left(\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right) = \frac{P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)}{P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)}\cdot P\left(\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right). \quad (8)$$

Assuming the priori probabilities $P\left(\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)$ and $P\left(\hat{\mathbf{s}}'^{p_j^{(i)}+l^{(i)}+l_h-2}_{p_j^{(i)}}\right)$ are equiprobable, the ratio of the posteriori probabilities $P\left(\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)$ and $P\left(\hat{\mathbf{s}}'^{p_j^{(i)}+l^{(i)}+l_h-2}_{p_j^{(i)}}\,\middle|\,\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)$ in equation (7) can be expressed as the likelihood ratio. That is,

$$\hat{\mathrm{P}} = \underset{\text{all } p_{j\in\{1,\dots,L_i\}}^{i\in\{1,\dots,E\}}}{\arg\min}\left(\frac{P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)}{P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}'^{p_j^{(i)}+l^{(i)}+l_h-2}_{p_j^{(i)}}\right)} < 1\right). \quad (9)$$

In the case of the AWGN, the likelihood probability $P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right)$ in nominator is given by

$$P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\right) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{l^{(i)}+l_h-1}\exp\left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\frac{\left(r_k-\hat{s}_k\right)^2}{2\sigma^2}\right]. \quad (10)$$

Likewise, the likelihood probability $P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}'^{p_j^{(i)}+l^{(i)}+l_h-2}_{p_j^{(i)}}\right)$ in denominator is

$$P\left(\mathbf{r}_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\,\middle|\,\hat{\mathbf{s}}'^{p_j^{(i)}+l^{(i)}+l_h-2}_{p_j^{(i)}}\right) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{l^{(i)}+l_h-1}\exp\left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\frac{\left(r_k-\hat{s}'_k\right)^2}{2\sigma^2}\right]. \quad (11)$$

Then, with equations (10) and (11), equation (9) can be written as

$$\exp\left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\frac{\left(r_k-\hat{s}_k\right)^2}{2\sigma^2}\right] < \exp\left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\frac{\left(r_k-\hat{s}'_k\right)^2}{2\sigma^2}\right]. \quad (12)$$

After taking the natural logarithm, (12) becomes

$$\left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left(r_k-\hat{s}_k\right)^2\right] < \left[-\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left(r_k-\hat{s}'_k\right)^2\right]. \quad (13)$$

By rearranging equation (13),

$$\left[\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left(r_k-\hat{s}'_k\right)^2 - \sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left(r_k-\hat{s}_k\right)^2\right] < 0\,. \quad (14)$$

Hence, the criterion for finding the most likely error starting position among $\left\{p_j^{(i)},\ i=1,\dots,E \text{ and } j=1,\cdots,L_i\right\}$ for the prescribed error events $\left\{\mathbf{e}^{(i)},\ i=1,\cdots,E\right\}$ is

$$\hat{\mathrm{P}} = \underset{\text{all } p_{j\in\{1,\dots,L_i\}}^{i\in\{1,\dots,E\}}}{\arg\min}\left(\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(r_k-\hat{s}'_k\right)^2 - \left(r_k-\hat{s}_k\right)^2\right] < 0\right). \quad (15)$$

That is, equation (15) means that the error correction of the new scheme is only performed based on an error starting position and its error-type associated with minimum one when

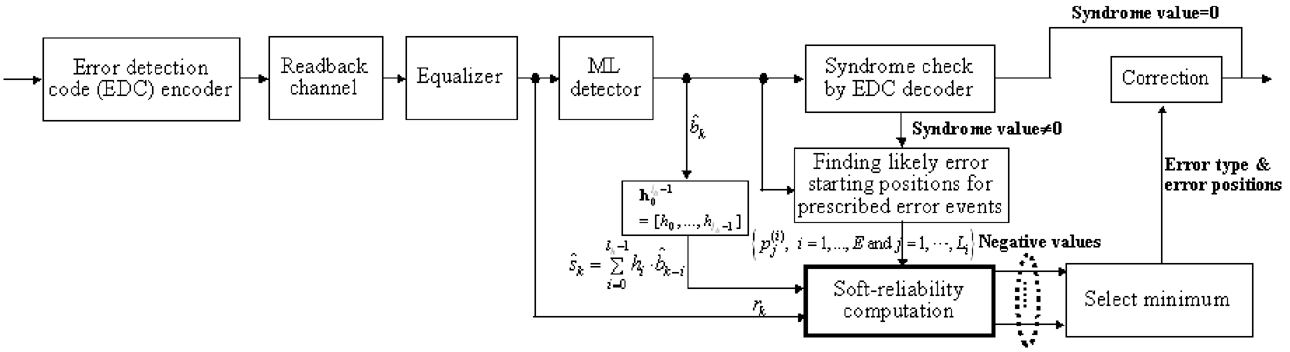


Fig. 1. Block diagram of soft-reliability information-based post-Viterbi processor.

it exists positions with negative soft-reliability value among all likely error starting positions for prescribed error events.

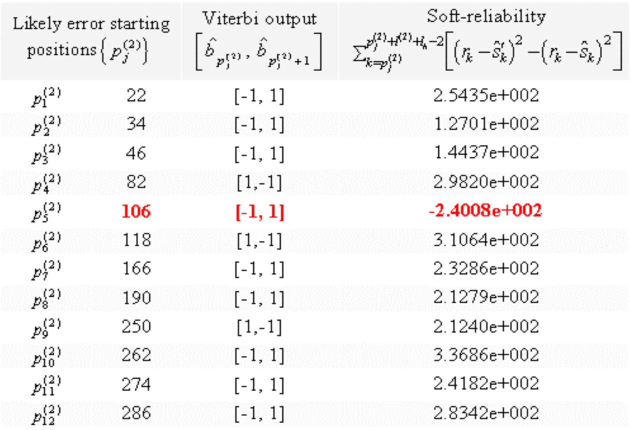
Figure 1 shows a generic soft-reliability information-based error correction scheme. In case the bolded block in Fig. 1 is replaced by the bank of error correlation filters, the post-Viterbi detector becomes almost the same as an error correlation filter-based post-Viterbi processor [4]. In essence, the procedure for finding likely error starting positions is the same as error correlation filter-based error correction scheme [4]-[12]. The main difference between the two schemes, that is, soft-reliability information-based and error correlation filter-based post-Viterbi processors, is how they find the most probable error starting position. Among all likely error starting positions for prescribed error events, the new scheme selects an error starting position and its error-type that yields a minimum negative soft-reliability estimate, while the conventional scheme chooses an error starting position and its error-type that produces a maximum correlation value. When the syndrome of the codeword estimated by the ML detector is non-zero, the new scheme is activated. Based on the syndrome, the new scheme generates a set of likely error starting positions corresponding to each prescribed error event and then it attempts to find a position and its error-type satisfying equation (15). If a position satisfying equation (15) does not exist, the new scheme does not make a correction. However, the conventional scheme will make a correction because it does not have a criterion for judgment whether an estimated position is correctly determined, and consequently, this will result in miss-correction resulting in an increased number of errors. Suppose that a non-prescribed error event occurs in a codeword. The conventional scheme corrects one of the prescribed error events associated with a position, which yields the maximum correlation value among the given all likely positions for prescribed error events. As a result, the number of bit errors increases which can be a significant detrimental factor to the detection performance. The new scheme, however, attempts to correct an error event only if its soft-reliability estimate is negative. So, although a non-prescribed error event occurs in a codeword, the new scheme does not try to correct it because its soft-reliability estimate is hardly negative. Accordingly, based on the criterion that error correction should be only performed when a position with negative soft-reliability estimate exists, the new scheme yields low probability of miss-correction compared to the conventional scheme.

Table 1 shows an example of an error correction by the new scheme, which is based on a (300, 294) cyclic-redundancy check (CRC) code [11][13] with a generator polynomial $g(x) = 1+x+x^2+x^4+x^5+x^6$. The result is obtained through simulation when the syndrome value in decimal number of the estimated codeword is 43, and an actual error event and its error starting position are $\mathbf{e}^{(2)} = [2, -2]$ with length $l^{(2)} = 2$ and 106, respectively, and the number of prescribed error events E is 13. In our simulations, the channel is modeled as AWGN with channel target response $\mathbf{h}_0^3 = [1, 6, 7, 2]$ with $l^h = 4$ ($\mathbf{h}(D) = 1 + 6D + 7D^2 + 2D^3$), where D is one-symbol delay. In Table 1(a), the first column is likely error starting positions $\left\{p_j^{(2)},\ j = 1, \cdots, L_2 (= 12)\right\}$ associated with an occurred error event $\mathbf{e}^{(2)} = [2, -2]$ generated from syndrome value 43, and the second column shows the output bits of the ML detector with $l^{(2)} = 2$ $\left\{(\hat{b}_{p_j^2}, \hat{b}_{p_j^2+1}),\ j = 1, \cdots, 12\right\}$ starting at $\left\{p_j^{(2)},\ j = 1, \cdots, 12\right\}$, and the last column shows the soft-reliability values $\{\sum_{k=p_j^{(i)}}^{p_j^{(2)}+4} \left[\left(r_k - \hat{s}'_k\right)^2 - \left(r_k - \hat{s}_k\right)^2 \right], j = 1, \ldots, 12\}$ at $\left\{p_j^{(2)},\ j = 1, \cdots, 12\right\}$ corresponding to $\mathbf{e}^{(2)} = [2, -2]$. In the example, among $\left\{p_j^{(2)},\ j = 1, \cdots, 12\right\}$, the soft-reliability estimate is only negative at the error starting position $p_5^{(2)} = 106$ of an occurred error event $\mathbf{e}^{(2)} = [2, -2]$. Soft-reliability values for prescribed error events $\left\{\mathbf{e}^{(i)},\ i = 1, \cdots, E (= 13) \text{ and } i \neq 2\right\}$ are not shown in the paper, but we identified that they are all positive over the corresponding likely error starting positions $\left\{p_j^{(i)},\ i = 1, \ldots, 13 (i \neq 2) \text{ and } j = 1, \cdots, L_i\right\}$. Accordingly, an error starting position and its error event satisfying equation (15) become $p_5^{(2)} = 106$ and $\mathbf{e}^{(2)} = [2, -2]$, respectively. Finally, based on $\mathbf{e}^{(2)} = [2, -2]$ and $p_5^{(2)} = 106$, the new scheme performs error correction. As a reference, detail parameter

TABLE 1

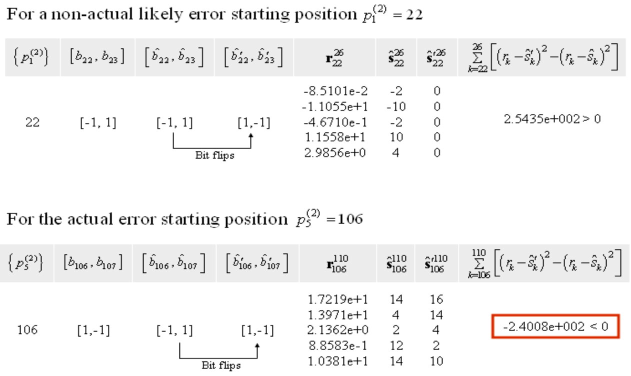
AN EXAMPLE OF ERROR CORRECTION OF THE NEW SCHEME

(a) SOFT-RELIABLITY COMPUTATION



Likely error starting positions $\{p_j^{(2)}\}$		Viterbi output $\left[\hat{b}_{p_j^{(2)}}, \hat{b}_{p_j^{(2)}+1}\right]$	Soft-reliability $\sum_{k=p_j^{(2)}}^{p_j^{(2)}+l^{(2)}+l_k-2}\left[\left(r_k-\hat{s}'_k\right)^2-\left(r_k-\hat{s}_k\right)^2\right]$
$p_1^{(2)}$	22	[-1, 1]	2.5435e+002
$p_2^{(2)}$	34	[-1, 1]	1.2701e+002
$p_3^{(2)}$	46	[-1, 1]	1.4437e+002
$p_4^{(2)}$	82	[1,-1]	2.9820e+002
$p_5^{(2)}$	**106**	**[-1, 1]**	**-2.4008e+002**
$p_6^{(2)}$	118	[1,-1]	3.1064e+002
$p_7^{(2)}$	166	[-1, 1]	2.3286e+002
$p_8^{(2)}$	190	[-1, 1]	2.1279e+002
$p_9^{(2)}$	250	[1,-1]	2.1240e+002
$p_{10}^{(2)}$	262	[-1, 1]	3.3686e+002
$p_{11}^{(2)}$	274	[-1, 1]	2.4182e+002
$p_{12}^{(2)}$	286	[-1, 1]	2.8342e+002

(b) DETAIL PARAMETER VALUES FOR SOFT-RELIABLITY COMPUTATION AT TWO POSITIONS



For a non-actual likely error starting position $p_1^{(2)}=22$

$\{p_1^{(2)}\}$	$[b_{22}, b_{23}]$	$[\hat{b}_{22}, \hat{b}_{23}]$	$[\hat{b}'_{22}, \hat{b}'_{23}]$	$\mathbf{r}_{22}^{26}$	$\hat{\mathbf{s}}_{22}^{26}$	$\hat{\mathbf{s}}_{22}^{\prime 26}$	$\sum_{k=22}^{26}\left[\left(r_k-\hat{s}'_k\right)^2-\left(r_k-\hat{s}_k\right)^2\right]$
22	[-1, 1]	[-1, 1]	[1,-1]	-8.5101e-2 -1.1055e+1 -4.6710e-1 1.1558e+1 2.9856e+0	-2 -10 -2 10 4	0 0 0 0 0	2.5435e+002 > 0

Bit flips

For the actual error starting position $p_5^{(2)}=106$

$\{p_5^{(2)}\}$	$[b_{106}, b_{107}]$	$[\hat{b}_{106}, \hat{b}_{107}]$	$[\hat{b}'_{106}, \hat{b}'_{107}]$	$\mathbf{r}_{106}^{110}$	$\hat{\mathbf{s}}_{106}^{110}$	$\hat{\mathbf{s}}_{106}^{\prime 110}$	$\sum_{k=106}^{110}\left[\left(r_k-\hat{s}'_k\right)^2-\left(r_k-\hat{s}_k\right)^2\right]$
106	[1,-1]	[-1, 1]	[1,-1]	1.7219e+1 1.3971e+1 2.1362e+0 8.8583e-1 1.0381e+1	14 4 2 12 14	16 14 4 2 10	-2.4008e+002 < 0

Bit flips

values for soft-reliability computations at $p_1^{(2)}=22$ and $p_5^{(2)}=106$ are also given by Table 1 (b).

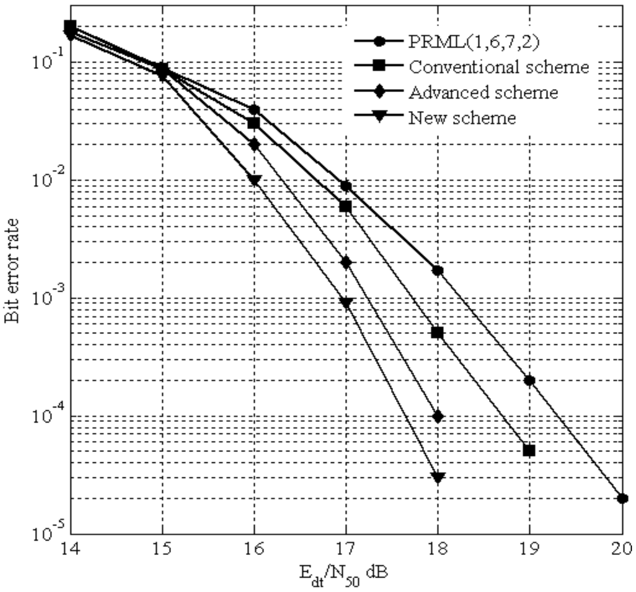


Fig. 2. Comparison of BERs of post-Viterbi processors under N_{50}.

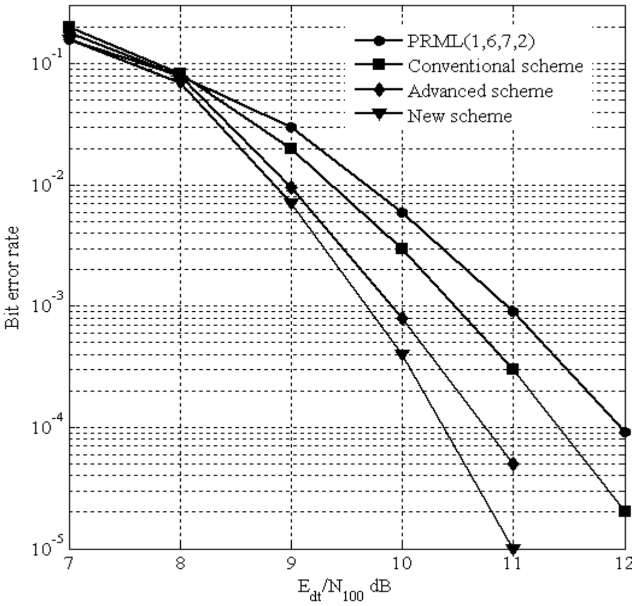


Fig. 3. Comparison of BERs of post-Viterbi processors under N_{100}.

III. SIMULATION RESULTS

A (203, 200) CRC code generated by a generator polynomial $g(x)$ = $1+x^2+x^3$ is used as an error detection code and the code can detect the dominant error events for perpendicular recording [11][12]. The bit error rates (BERs) of post-Viterbi processors are simulated and compared at user density D_u=1.4 with the channel target response of $\mathbf{h}(D)$ = $1 + 6D + 7D^2 + 2D^3$ over perpendicular recording. The user density D_u is defined by $D_u = R_{CRC} \times D_c$, where R_{CRC} is the code rate of the CRC code and D_c is the channel density. The dominant error events (E=6) at user density 1.4 are listed in [4][11][12]. As a reference, the BER of a PRML (1, 6, 7, 2) system is also shown at the same user density. The signal-to-noise ratio (SNR) has been defined in [12] as the ratio of the energy of the first derivative of the transition response E_{dt} and the noise spectral density. In our simulations, the noise parameter N_{50} or N_{100} in the SNR definition signifies a mixture of 50% AWGN and 50% jitter noise or 100% AWGN, respectively. Fig. 2 shows the BERs of conventional, advanced and new post-Viterbi processors under N_{50}. In Fig. 2, the legend “conventional scheme” [5]-[12] corresponds to an error correlation filter-based post-Viterbi processor where error starting positions for each error event are all positions in codeword block, and “advanced scheme”[4] means error correlation filter-based post-Viterbi processor that error starting positions for each error event are more reliable than those of “conventional scheme”, and “new scheme” is soft-reliability information-based post-Viterbi processor, which is proposed in the paper. It can be seen that post-Viterbi processors produce considerable performance gains compared to conventional PRML (1, 6, 7, 2) system. Among the post-Viterbi processors, the “conventional scheme” performs worst because of the miss-correction, i.e., miss-selection and miss-positioning. In the "advanced scheme”, the miss-correction of the dominant error events is reduced due to the information about likely error starting positions. The “New scheme” yields the largest performance gain among the schemes considered here. As mentioned earlier, while the error correlation filter-based post-Viterbi processor does not have any criterion for judgment whether an estimated error starting position is really correct, the soft-reliability information-based post-Viterbi processor can judge the correctability of an estimated error starting position based on the soft-reliability information. Thus, given all likely error positions for given error events, if their soft-reliability estimates for all likely error starting positions are positive, then an attempt for correction is not made, which does not increase the number of bit errors. Thus, our simulation results indicate that the soft- reliability information-based error correction scheme performs better than the error correlation filter-based post-Viterbi processor. Fig. 3 shows a comparison of the BERs of conventional, advanced, and new post-Viterbi processors under N_{100} . The result shows the same performance trend as shown in Fig. 2, irrespective of the noise distribution.

IV. CONCLUSION

We have investigated a soft-reliability information-based post-Viterbi processor for reducing the miss-correction of error correlation filter-based post-Viterbi processor. In case error correction is only performed if a position with negative soft-reliability estimate exists, the new scheme performs significantly better than the conventional one. We conclude that the new technique is a good candidate for high-capacity storage systems.

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