Spectral Null Codes

K. A. SCHOUHAMER IMMINK, FELLOW, IEEE

***Abstract*—The servo position information of magnetic tape or disk recorders is often recorded as low-frequency components usually called pilot tracking tones. Binary codes giving rise to a spectral null at an arbitrary frequency are used to provide space for the allocation of auxilliary pilot tones. Encoding methods are treated in which binary data are mapped into constrained binary sequences for shaping the spectrum. The rate and power spectral density function of memoryless codes that exhibit spectral nulls are computed. The relationship between code redundancy and spectral notch width is quantified with a parameter called the "sum variance."**

I. Introduction

An IMPORTANT class of codes found in recording systems are spectral null codes, that is, codes that have zero power spectral density at specific frequencies. Among these, codes with a spectral null at $f = 0$, also called dc-free or dc-balanced codes, have predominated in applications. The field of application of digital channel codes with suppressed low-frequency components is quite broad. We find applications in transmission systems over fiber or metallic cable and in storage media such as magnetic or optical recording. Codes with spectral nulls at frequencies other than zero are often used to provide space for pilot tones. For example, in magnetic disk files information is written on and read from concentric tracks on the disks. At typical state-of-the-art track densities, of, say, ten tracks/mm, such a disk file is usually provided with position reference information employed by a head-positioning servo system to position and maintain the head precisely over a selected track. The operation of maintaining the head on the track is known as track following. In some disk files, position reference information is provided remotely from a dedicated servo disk [1]. In drives with a small number of platters, the use of an entire surface for servo information gives an excessive loss of data-recording area. At higher track densities, such an arrangement has the extra disadvantage that it is difficult to guarantee alignment between the remote servo disk and the actual information disk. To overcome these difficulties, various techniques to provide position information from the information disk itself have been proposed and implemented. This virtually eliminates thermal effects on head positioning and allows great increases in storage density. Often, the servo position information is recorded as low-frequency conponents, usually called pilot tracking tones. In a typical embodiment, the servo position information consists of two (or more) single-frequency signals recorded deeply, or buried, in the magnetic medium below the area used for user information [2], [3]. It should be appreciated that the pilot tone is prerecorded once in a disk's lifetime and cannot be removed by a user. To circumvent interaction during readout between the buried pilot tones and the user information, user information is often encoded in such a way that the power spectral density function of the encoded stream vanishes at the pilot tone frequencies [4]. Coding techniques are required to provide this degree of freedom. The application of spectral null codes is not confined to magnetic recorders, their utilization in optical recording systems has been reported by Carasso and Huijser [5]. They implemented a timing signal at 1/2 the code clock frequency in the form of a groove depth modulation, leading to a null requirement at the Nyquist frequency.

The outline of the paper is as follows. We commence by counting the number of codewords that comply with specified spectral requirements, and we derive an expression for the power spectral density of randomly cascaded codewords. An important quantity used in the performance evaluation of codes with a null at the zero-frequency dc-balanced codes is the variance of the running digital sum, in short, *sum variance*. The concept of sum variance is extended to spectral codes with spectral nulls at frequencies other than zero.

II. Spectral Nulls

Let $\boldsymbol{x} = (x_1, \cdots, x_n)$, $x_i \in \{-1, 1\}$, be an element of a set S of codewords. The discrete Fourier transform $X(\omega)$ of the codeword $\boldsymbol{x}$ is given by [6]

$$X(\omega) = \sum_{i=1}^{n} x_i e^{-ji\omega}, \qquad -\pi \leq \omega \leq \pi \tag{1}$$

where $j = \sqrt{-1}$. For all i, $1 \leq i \leq n$, let the number of codewords $\boldsymbol{x}$ in S with $x_i = 1$ be equal to the number of codewords with $x_i = -1$ (i.e., the sum of all codewords in S is the all-zero vector). If, in addition, codewords are randomly chosen from S to form an infinite sequence, then it is rather straightforward to show, following [7] and [8], that the power spectral density function $H(\omega)$ of the concatenated sequence, when transmitted serially, is

$$H(\omega) = \frac{1}{Mn} \sum_{i=0}^{M-1} \left| X^{(i)}(\omega) \right|^2, \tag{2}$$

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The author is with the Philips Research Laboratories, 5600 JA Eindhoven, The Netherlands.
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where $X^{(i)}(\omega)$ is the discrete Fourier transform of the ith element of S and M is the cardinality of S. Note that, due to our assumptions M has to be even. For the sake of convenience, we have assumed that the codewords are equiprobable. This restriction, however, is not necessary and can be relaxed (cf. [7] and [8]).

For a sequence of length n to have a null at the frequency $f\,(=\omega/2\pi) = 1/k$, k an integer, it is sufficient to satisfy $|X(2\pi/k)| = 0$, or

$$\sum_{i=1}^{n} x_i e^{-j2\pi i/k} = 0. \quad (3)$$

As was remarked by Gorog [6], the identity

$$e^{-j2\pi i/k} = e^{-j2\pi(i+k)/k}$$

indicates that one may construct a desired sequence by balancing k-vectors rather than n-vectors. If, for the time being, we restrict the codeword length n to an integral multiple of k, i.e.,

$$n = ks, \quad (4)$$

then we obtain

$$\sum_{i=1}^{n} x_i e^{-j2\pi i/k} = \sum_{m=1}^{k} A_m e^{-j2\pi m/k} = 0 \quad (5)$$

where

$$A_i = \sum_{m=0}^{s-1} x_{mk+i}, \qquad i = 1, \cdots, k.$$

If k is a prime number, (5) can only be satisfied if (the proof is given in [4])

$$A_1 = A_2 = \cdots = A_k.$$

As an example, if $k = 3$ and $n = 6$, the following relationship must hold:

$$A_1 = A_2 = A_3$$

$$x_1 + x_4 = x_2 + x_5 = x_3 + x_6.$$

At this point it is appropriate to consider the following property. As previously remarked, (5) can only be satisfied provided

$$A_1 = A_2 = \cdots = A_k$$

or, in this case,

$$\sum_{i=1}^{n} x_i e^{-j2\pi i/k} = \sum_{l=1}^{k} A_l e^{-j2\pi l/k} = A_1 \sum_{l=1}^{k} e^{-j2\pi l/k} = 0.$$

Since

$$\sum_{l=1}^{k} e^{-j2\pi lm/k} = 0, \ \gcd(m, k) = 1,$$

we conclude that a spectral null at frequency $1/k$ implies spectral nulls at frequencies m/k, $\gcd(m, k) = 1$.

Essentially, the codewords of length n consist of k interleaved subwords of length s. For convenience, we define the sets

$$S_i = \{i, i + k, i + 2k, \cdots, i + (s - 1)k\},$$
$$1 \le i \le k.$$

Rewriting the expression of A_i yields

$$A_i = \sum_{m \in S_i} x_m, \qquad i = 1, \cdots, k.$$

Since the A_i' s are each the sum of s binary elements, they can take only a limited number of different values, namely, $A_i \in \{-s, -s + 2, \cdots, s - 2, s\}$. To find the number of codewords satisfying (3), denoted by $M_{n,k}$, k a prime number, is now a matter of counting:

$$M_{n,k} = \sum_{i=0}^{s} \binom{s}{i}^k. \quad (6)$$

In Table 1, we have collected some values of $M_{n,k}$ for various values of the codeword length n and k.

The rate R of the code is defined as

$$R = \frac{1}{n} \log_2 M_{n,k}. \quad (7)$$

The quantity, called sum variance $s^2_{1/k}$, will be explained in a moment.

A. Computation of the Spectrum

In this section, we derive an expression for the power spectral density function. We shall not use (2) but a more convenient alternative. If the codewords are chosen with equal probabilities form S to form an infinite sequence, then the autocorrelation function of the cascaded sequence, denoted by μ_m, $-(n-1) \le m \le n-1$, is

$$\mu_{-m} = \mu_m = \frac{1}{nM_{n,k}} \sum_{i=1}^{n-m} \sum_{j=0}^{M_{n,k}-1} x_i^{(j)} x_{i+m}^{(j)},$$
$$1 \le m \le n - 1 \quad (8)$$

where $x_i^{(j)}$ denotes the ith symbol of the jth codeword in S. Outside the interval $-(n-1) \le m \le n-1$, we find $\mu_m = 0$. Since $x_i \in \{-1, 1\}$ it is obvious that $\mu_0 = 1$. The power spectral density expressed in terms of the autocorrelation function is

$$H_{n,k}(\omega) = \mu_0 + 2 \sum_{m=1}^{n-1} \mu_m \cos m\omega. \quad (9)$$

As mentioned in the previous section, the codewords consist of k interleaved subwords of length s. For symmetry reasons, we find that the correlation

$$\sum_{j=0}^{M_{n,k}-1} x_{j_1}^{(j)} x_{j_2}^{(j)}$$

between symbols does not depend on the absolute positions j_1 and j_2, $j_1 \neq j_2$ but solely on whether or not j_1 and j_2 are in different subwords.

Let ρ_0 and ρ_1 denote the correlation between symbols within or outside the same subword, respectively; we then

TABLE I
NUMBER OF CODEWORDS VERSUS THE CODEWORD LENGTH n AND SPECTRAL NULL $1/k$

k	s	n	$M_{n,k}$	R	$s^2_{1/k}$
2	2	4	6	0.646	0.833
2	3	6	20	0.720	1.167
2	4	8	70	0.766	1.500
2	5	10	252	0.798	1.833
3	2	6	10	0.554	1.333
3	3	9	56	0.645	1.810
3	4	12	346	0.703	2.320
3	5	15	2252	0.742	2.822
5	2	10	34	0.509	2.438
5	3	15	488	0.595	3.117
5	4	20	9826	0.663	4.024
5	5	25	206 252	0.706	4.823
7	2	14	130	0.502	3.487
7	3	21	4376	0.576	4.442
7	4	28	312 706	0.652	5.750
7	5	35	20 156 252	0.693	6.815

conclude that

$$\rho_0 = \frac{1}{Mn} \sum_{j=0}^{M-1} x_{j_1}^{(j)} x_{j_2}^{(j)}, \qquad j_1, j_2 \in S_i, j_1 \neq j_2$$

and

$$\rho_1 = \frac{1}{Mn} \sum_{j=0}^{M-1} x_{j_1}^{(j)} x_{j_2}^{(j)}, \qquad j_1 \in S_i, j_2 \notin S_i$$

where $M = M_{n,k}$ is used for clerical convenience. The autocorrelation function is of the form

$$\mu_{-m} = \mu_m = \begin{cases} 1, & m = 0 \\ (n-m)\rho_1, & 0 < m \leq n, m \neq k, 2k, \cdots \\ (n-m)\rho_0, & m = k, 2k, \cdots, (s-1)k \\ 0, & m > n \end{cases} \tag{10}$$

where ρ_0 and ρ_1 are constants to be determined in a moment. So that, after a routine computation, we obtain the following expression of the power spectral density function:

$$H_{n,k}(\omega) = 1 - n\rho_0 + \rho_1 \left(\frac{\sin n\omega/2}{\sin \omega/2}\right)^2 + k(\rho_0 - \rho_1)\left(\frac{\sin n\omega/2}{\sin k\omega/2}\right)^2. \tag{11}$$

As a direct consequence of $H_{n,k}(2\pi/k) = 0$, we find the following relationship between ρ_0 and ρ_1:

$$1 - n\rho_0 + \frac{n^2}{k}(\rho_0 - \rho_1) = 0, \qquad k \neq 1. \tag{12}$$

B. Computation of the Correlation

We examine now the computation of the correlation ρ_0 of symbols at positions j_1 and j_2, $j_1, j_2 \in S_i$, within one subword, or

$$\rho_0 = \frac{1}{Mn} \sum_{j=0}^{M-1} x_{j_1}^{(j)} x_{j_2}^{(j)}, \qquad j_1 \neq j_2. \tag{13}$$

Keeping in mind that $x_i \in \{-1, 1\}$ results in

$$\begin{aligned}\rho_0 &= \frac{1}{Mn} \sum_{j=0}^{M-1} x_{j_1}^{(j)} x_{j_2}^{(j)} \\ &= \frac{1}{Mn}\left\{N(x_{j_1}x_{j_2} = 1) - N(x_{j_1}x_{j_2} = -1)\right\} \\ &= \frac{1}{n}\left\{\frac{2}{M} N(x_{j_1} = x_{j_2}) - 1\right\} \\ &= \frac{1}{n}\left\{\frac{4}{M} N(x_{j_1} = x_{j_2} = 1) - 1\right\}, \qquad j_1 \neq j_2 \end{aligned} \tag{14}$$

where $N(\cdot)$ is the number of codewords satisfying $(\cdot)$. By inspection, we find that the number of codewords for which $x_{j_1} = x_{j_2} = 1$ equals

$$N(x_{j_1} = x_{j_2} = 1) = \sum_{i=2}^{s} \binom{s-2}{i-2} \binom{s}{i}^{k-1}. \tag{15}$$

Equations (11), (12), and (15) provide sufficient information to compute easily the power spectral density function. Results of calculations are displayed in Fig. 1, which shows the spectra of codes with a spectral null at $k = 3$ with the codeword length n as a parameter. Similar curves for $k = 5$ and $k = 7$ are shown in Figs. 2 and 3, respectively. It can be seen in the plots that the higher the rate of the code, the smaller the width of the spectral notch. This, in fact, reflects what we intuitively expect, and it illustrates very well that for a wider frequency region of suppressed components one has to pay more in terms of code redundancy. In the next sections, we take a closer look at these effects and attempt to quantify the relationship between code redundancy and spectral notch width.

C. Sum Variance

Before we proceed with our analysis, it is helpful to summarize some of the properties of codes with a spectral null at dc. In the analysis and synthesis of codes with suppressed low-frequency components, usually termed dc-free codes, the running digital sums is of prime importance. Consider a sequence of two-level symbols $\{\cdots, x_{-1}, x_0, x_1, x_2, \cdots\}$. The running digital sum, denoted by z_j, is defined as

$$z_j = \sum_{i=-\infty}^{j} x_i. \tag{16}$$

As discussed in [9], a sequence is dc-free if the running digital sum is bounded. An important quantity used in the performance evaluation of dc-balanced codes is the variance of the running digital sum, in short, sum variance. The sum variance of a dc-free sequence is defined as

$$s_0^2 = E_i\{z_i^2\}, \tag{17}$$

where $E\{\cdot\}$ denotes the expectation operator.

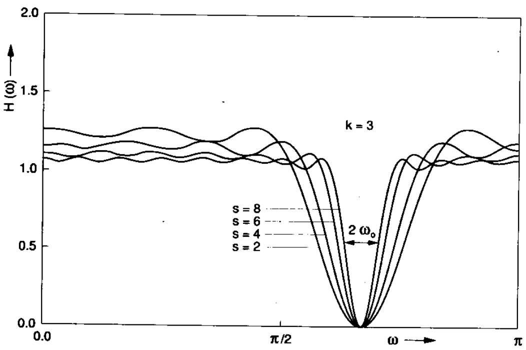

Fig. 1. Power spectral density function versus frequency with codeword length n as parameter. Spectral null at 1/3. Width of spectral notch has been indicated in figure.

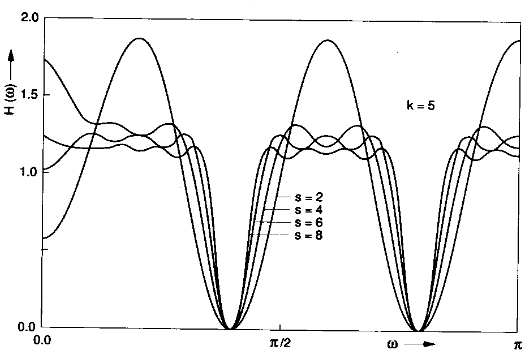

Fig. 2. Power spectral density function versus frequency with codeword length n as parameter. Spectral null at 1/5.

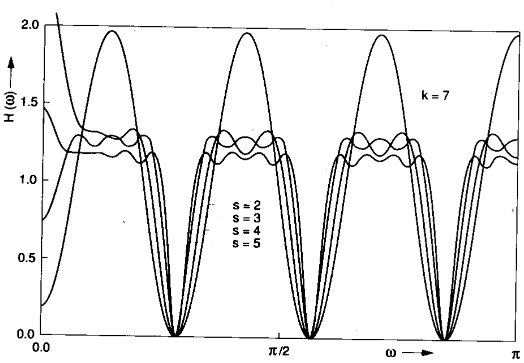

Fig. 3. Power spectral density function versus frequency with codeword length n as parameter. Spectral null at 1/7.

Let $H_0(\omega)$ denote the power density function of a sequence with vanishing power at dc. The width of the spectral notch is commonly quantified by a parameter called the *cutoff frequency*. The cutoff frequency ω_0 of a dc-constrained sequence is defined by [10], [11]

$$H_0(\omega_0) = 1/2. \tag{18}$$

Justesen [10] derived an approximation of the cutoff frequency ω_0 in terms of the sum variance:

$$2s_0^2\omega_0 \simeq 1. \tag{19}$$

Thus the cutoff frequency of dc-balanced codes is proportional to the inverse of the variance of the running digital sum. This simple relation is not limited in validity to the dc-balanced sequences with an exponentially decaying autocorrelation function for which it was derived but applies to sequences generated by implemented channel codes as well. Detailed computations of examples of implemented channel codes made by Justesen [10] and Immink [12] show that the former relation (19) between cutoff frequency and sum variance is quite accurate. This result has motivated us to apply the sum variance as a criterion of the spectral properties of a channel code, which is of practical importance since the sum variance of a sequence can often be evaluated by simple calculations even though the autocorrelation function and corresponding spectrum are complicated functions.

In similar vein to the case of dc-balanced codes, the 'running sum' concept has been extended by Marcus and Siegel [4] to codes with spectral nulls at $f = 1/k$. The (complex) running sum of a sequence with a spectral null at the frequency $f = 1/k$ is defined as

$$z_{m,f} = \sum_{i=-\infty}^{m} x_i e^{-j2\pi i/k}. \tag{20}$$

Similarly, the running sum variance of the sequence is defined as

$$s_f^2 = E_i\left\{\left|z_{i,f}\right|^2\right\}. \tag{21}$$

Specifically, the running sum variance of a sequence composed of cascaded memoryless n sequences is simply the arithmetic average of the sum variance of each codeword, or

$$s_{1/k}^2 = \frac{1}{nM}\sum_{i=1}^{n}\sum_{m=0}^{M-1}\left|x_i^{(m)}e^{-j2\pi i/k}\right|^2. \tag{22}$$

Naturally, this operation of enumerating all codewords is, for large codeword sets, a considerable computational load, but fortunately, the sum variance can be conveniently expressed in terms of the autocorrelation coefficients μ_i. Consider to this end the variable

$$z_{m,f} - z_{0,f} = x_1 e^{-j2\pi/k} + x_2 e^{-j2\pi 2/k} + \cdots + x_m e^{-j2\pi m/k}.$$

Taking the variance of this variable yields

$$\begin{aligned} 2s_f^2 &- 2E_i\left\{z_{i,f}z_{i+m,f}\right\} \\ &= \sum_{j=-m+1}^{m-1}(m - |j|)\,\mu_j\cos 2\pi j/k \\ &= m\sum_{j=-m+1}^{m-1}\mu_j\cos 2\pi j/k - 2\sum_{j=1}^{m-1} j\mu_j\cos 2\pi j/k. \end{aligned}$$

When we take the limit for $m \to \infty$ and use

$$\lim_{m\to\infty} E_i\left\{z_{i,f}z_{i+m,f}\right\} = 0$$

and

$$\lim_{m\to\infty}\sum_{j=-m+1}^{m-1}\mu_j\cos 2\pi j/k = H_{n,k}(2\pi/k) = 0,$$

we obtain

$$s_f^2 = -\sum_{i=1}^{\infty} i\mu_i \cos 2\pi i/k. \quad (23)$$

Given the autocorrelation coefficients of the memoryless block encoded sequence, we obtain, after substitution of (10), the sum variance at the frequency $1/k$

$$s_{1/k}^2 = -\rho_1 \sum_{i=1}^{n-1} i(n-i) \cos 2\pi i/k - (\rho_0 - \rho_1) \cdot k^2 \sum_{i=1}^{s-1} i(s-i). \quad (24)$$

The preceding expression can be simplified using the identities

$$\sum_{i=1}^{n-1} i \cos 2\pi i/k = -\frac{n}{2}$$

$$\sum_{i=1}^{n-1} i^2 \cos 2\pi i/k = -\frac{n^2}{2} + \frac{n}{2\sin^2 \pi/k},$$

from which we obtain

$$s_{1/k}^2 = \rho_1 \frac{n}{2\sin^2 \pi/k} + \frac{1}{6}(\rho_1 - \rho_0) k^2 s(s^2 - 1). \quad (25)$$

We may therefore determine the sum variance $s_{1/k}^2$ without explicitly calculating the sum variance of each codeword. Numerical results are provided in Table I. As already remarked, a spectral null at $1/k$ implies spectral nulls at its harmonics m/k, gcd $(m, k) = 1$. Similarly, we may obtain the sum variance at the spectral null m/k, gcd $(m, k) = 1$

$$s_{m/k}^2 = \rho_1 \frac{n}{2\sin^2 \pi m/k} + \frac{1}{6}(\rho_1 - \rho_0) k^2 s(s^2 - 1). \quad (26)$$

Using the preceding equations, a numerical analysis showed, for the range of parameters listed in Table I, that the difference between the sum variance at the spectral nulls m/k, $m \neq 1$, and the sum variance at the principal spectral null $1/k$ is slight (less than 1%).

The width of the spectral notch is an important quantity since it specifies the frequency region of suppressed components around the spectral null. The width of the spectral notch is defined in a similar way as was previously done for codes with a spectral null at the zero frequency by a quantity termed the cutoff frequency. The cutoff frequency, denoted by ω_0, is defined by (see also Fig. 1)

$$H(2\pi/k - \omega_0) = 1/2. \quad (27)$$

As previously discussed, dc-free sequences possess the property that the product of cutoff frequency ω_0 and the sum variance of the sequence s_0^2 is approximately $1/2$. It is conjectured here that this simple relationship not only holds for dc-free sequences, but applies as well to sequences with spectral nulls at other frequencies than the zero. In other words, we conjecture that

$$2s_{1/k}^2 \omega_0 \simeq 1. \quad (28)$$

The validity of this conjecture was numerically tested for the code parameters shown in Table I. It has been found that this relationship between the sum variance and the actual cutoff frequency is accurate within a few percent.

III. Spectrum Null at the Nyquist Frequency, $k = 2$

Codes with zero spectrum power at the Nyquist frequency $f = 1/2$ have a special structure that allows an easy computation of its properties. The number of codewords follows from (6)

$$M_{n,2} = \sum_{i=0}^{n/2} \binom{n/2}{i}^2 = \binom{n}{n/2}.$$

After some arithmetic, utilizing (14) and (15), we obtain

$$\rho_0 = -\rho_1 = -\frac{1}{n(n-1)},$$

so that the autocorrelation function is

$$\mu_{-m} = \mu_m = \begin{cases} 1, & m = 0 \\ (-1)^{m+1} \dfrac{(n-m)}{n(n-1)}, & 0 < m \leq n \\ 0, & m > n. \end{cases}$$

The corresponding power spectral density function is given by

$$H_{n,2}(\omega) = 1 - n\rho_0 - \rho_0 \left(\frac{\sin n\omega/2}{\sin \omega/2}\right)^2 + 4\rho_0 \left(\frac{\sin n\omega/2}{\sin \omega}\right)^2 = \frac{n}{n-1}\left\{1 - \left(\frac{\sin n\omega/2}{n \cos \omega/2}\right)^2\right\}.$$

The sum variance is

$$s_{1/2}^2 = \sum_{i=1}^{n-1} \frac{i(n-i)}{n(n-1)} = \frac{n+1}{6}.$$

We may observe a nice resemblance between the spectral properties of the memoryless code with a null at the Nyquist frequency and the memoryless code with a null at the zero frequency.

IV. Interleaving of Sequences

Interleaving of sequences is a common technique that is often utilized in error-correcting codes for bursty or diffuse channels. Prior to transmission the ordering of a sequence is rearranged in a deterministic manner so that a burst of errors is distributed more uniformly at the input of the decoder so that they can be corrected by a randomerror-correcting code. The interleaving structure can also fruitfully be applied to spectral shaping codes. Consider S_b, a set of codewords of length n that have a null at $f = 1/k$. Let $x^{(1)} \in S_b, \cdots, x^{(L)} \in S_b$. The L-way interleaved sequence $y = \{ y_1, \cdots, y_{1n} \}$ defined by

$$\begin{aligned} y &= \{ y_1, \cdots, y_{1n} \} \\ &= \{ x_1^{(1)}, x_1^{(2)}, \cdots, x_1^{(L)}, x_2^{(1)}, \cdots, x_2^{(L)}, \\ &\quad x_n^{(1)}, \cdots, x_n^{(L)} \} \end{aligned}$$

has simultaneous spectral nulls at frequencies $f = i/(kL)$, $i = 1, 2, \cdots$. If the autocorrelation function and spectrum of the basic code S_b are denoted by $\mu_{b,i}$ and $H_b(\omega)$, respectively, then the autocorrelation function of randomly cascaded L-way interleaved sequences is

$$\mu_i = \begin{cases} 0, & i \bmod L \neq 0 \\ \mu_{b,i/L}, & i \bmod L = 0. \end{cases} \tag{29}$$

The corresponding spectrum of the interleaved sequence is

$$\begin{aligned} H(\omega) &= 1 + 2 \sum_{m=1}^{\infty} \mu_m \cos m\omega \\ &= 1 + 2 \sum_{m=1}^{\infty} \mu_{b,Lm} \cos Lm\omega = H_b(L\omega). \end{aligned} \tag{30}$$

From (30), if $H_b(\omega_1) = 0$, then the spectrum of the L-way interleaved sequence has a null at ω_1/L, or $H(\omega_1/L) = 0$. Similarly, if the sum variance at frequency ω_1 of the basic code is $s_{\omega 1}^2$, then the sum variance at frequency ω_1/L of the L-way interleaved sequence is $Ls_{\omega 1}^2$.

Example 1: We examine an interleaved biphase code. The codewords of the basic biphase code are $\{ -1, 1 \}$ and $\{ 1, -1 \}$, and its spectrum is given by

$$H_2(\omega) = 2 \sin^2 (\omega/2). \tag{31}$$

A two-way interleaved biphase code has four codewords, namely $\{ -1, -1, 1, 1 \}$, $\{ 1, -1, -1, 1 \}$, $\{ 1, 1, -1, -1 \}$, and $\{ -1, 1, 1, -1 \}$. This code is commonly called *quad-phase*, and its spectrum is given by

$$H(\omega) = H_2(2\omega) = 2 \sin^2 \omega. \tag{32}$$

It can easily be verified that the quad-phase code has spectral nulls at both the zero and the Nyquist frequency.

V. Conclusion

We have computed the efficiency and spectral properties of memoryless block codes that exhibit a spectral null. The relationship between code redundancy and spectral notch width has been quantified with a parameter called the sum variance. We have found that twice the product of the spectral notch width and the sum variance is approximately unity.

References

[1] R. K. Oswald, "Design of a disk file head-positioning servo," *IBM J. Res. Develop.*, vol. 18, pp. 506-512, Nov. 1974.

[2] M. K. Haynes, "Magnetic recording techniques for buried servos," *IEEE Trans. Magn.*, vol. MAG-17, pp. 2730-2734, Nov. 1981.

[3] N. H. Hansen, "A head-positioning system using buried servos," *IEEE Trans. Magn.*, vol. MAG-17, pp. 2735-2738, Nov. 1981.

[4] B. H. Marcus and P. H. Siegel, "On codes with spectral nulls at rational submultiples of the symbol frequency," *IEEE Trans. Inform. Theory*, vol. IT-33, pp. 557-568, July 1987.

[5] M. G. Carasso and A. Huijser, "Principles of optical recording systems with AlGaAs lasers and pregrooved discs," in *Proc. Conf. Lasers and Electro-Optics*, Phoenix, AZ, 1982, p. 30.

[6] E. Gorog, "Redundant alphabets with desirable frequency spectrum properties," *IBM J. Res. Develop.*, vol. 12, pp. 234-241, May 1968.

[7] B. S. Bosik, "The spectral density of a coded digital signal," *Bell Syst. Tech. J.*, vol. 51, pp. 921-932, Apr. 1972.

[8] G. L. Pierobon, "Codes for zero spectral density at zero frequency," *IEEE Trans. Inform. Theory*, vol. IT-30, pp. 435-439, Mar. 1984.

[9] P. H. Siegel, "Recording codes for digital magnetic storage," *IEEE Trans. Magn.*, vol. MAG-21, no. 5, pp. 1344-1349, Sept. 1985.

[10] J. Justesen, "Information rate and power spectra of digital codes," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 457-472, May 1982.

[11] J. N. Franklin and J. R. Pierce, "Spectra and efficiency of binary codes without dc," *IEEE Trans. Commun.*, vol. COM-20, pp. 1182-1184, Dec. 1972.

[12] K. A. S. Immink, "Performance of simple binary dc-constrained codes," *Philips J. Res.*, vol. 40, pp. 1-21, 1985.

K. A. Schouhamer Immink (M'81-SM'86-F'90) was born in Rotterdam, The Netherlands, on December 18, 1946. He received the B.S. degree from the Rotterdam Polytechnic and the M.S. and Ph.D. degrees from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 1967, 1974 and 1985, respectively, all in electrical engineering.

He joined the Philips' Research Laboratories, Eindhoven, in 1968, where his work involved the signal processing side of optical recording systems. He was, among others, responsible for the design and development of channel coding techniques for the compact disk, compact disk video, and experimental erasable optical audio disks. Since 1986, he has been a member of the magnetic recording group at the Philips' Research. He has written numerous papers in the field of coding techniques for optical and magnetic recording and is co-author of the book, *Principles of Optical Disc Systems.* He holds more than 30 patents, mainly in the area of optical recording.

Dr. Immink was awarded a Fellowship by the Audio Engineering Society in 1985 for 'his work in the area of optical laser disc and for detailed study on channel codes for the compact disk'. He is a Fellow of the Institution of Electrical Engineers.