Advanced Soft-Reliability Information-Based Post-Viterbi Processor

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Abstract — *This paper proposes a new soft-reliability information-based post-Viterbi processor with advanced noise-robustness for reducing probability of miss-correction and no correction of a conventional soft-reliability-based post-Viterbi processor. Among all likely error starting positions for prescribed error events, the two schemes are equal to attempt to correct error-type corresponding to a position with minimum one only if there exist positions where a soft-reliability estimate is negative. The main difference between the two schemes is how they acquire the soft-reliability estimate. The soft-reliability estimate of the new scheme is obtained through the elimination of the noise-sensitive component from the log-likelihood ratio of the posteriori probabilities, which is the soft-reliability estimate of conventional scheme. As a result, the new scheme is based on more reliable soft-reliability information so reducing the probability of miss-correction and no correction.*

Index Terms — Error detection code, dominant error event, soft-reliability information-based post-Viterbi processor, cyclic redundancy check code.

I. INTRODUCTION

The demand for high-density digital data storage systems has been growing steadily. Although technological innovations in the design of recording media and heads are key to achieving high density recording systems, the role of sophisticated coding and signal processing techniques for data recovery is increasingly becoming crucial in supporting and augmenting these advancements.

There has been a growing interest in error detection codes with error correction properties [1]-[10]. Unlike conventional read channels, where the error correction code (ECC) is expected to correct all the errors at the output of the constrained decoder, dominant error events are corrected by applying a low redundancy error detection code. This error detection code is an inner ECC that can correct dominant error events at the output of the channel detector by using only a few parity bits. In this way, the correction capacity loss of the outer ECC is significantly reduced and the error propagation of the modulation decoder is also minimized. The approach using error detection codes, referred to as post-Viterbi processor (in other words, maximum likelihood (ML) post-processor), has found wide acceptance since the performance-complexity trade-off offered is very attractive and affordable. The above approach has been widely studied for magnetic recording channels, and for optical recording systems [1-10].

In conventional soft-reliability information-based post-Viterbi processor in conjunction with an error detection code [2], when the syndrome is non-zero, the error detection code generates a set of all likely error starting positions for prescribed error events. Then, the scheme computes the soft-reliability values over the set of likely error starting positions, and it outputs an error starting position and its error-type associated with minimum one only if there exist likely error starting positions that generate the negative soft-reliability estimate. Finally, based on the position and its error-type, the scheme performs the error correction. Note that the scheme attempts to perform error correction only if positions with negative soft-reliability estimate exist.

A soft-reliability estimate for the conventional scheme is given as the log-likelihood ratio of the posteriori probabilities [2], which includes a noise-sensitive component. The noise-sensitive component is the main source of detrimental factors such as miss-correction and no correction. For the actual error starting position, we found that the scheme leads to no correction or miss-correction because the soft-reliability estimate of the position fails to give negative value due to a specific noise pattern. Note that the soft-reliability for the actual error starting position usually gives a negative value, but sometimes a positive value is yielded only if the noise characteristic roughly matches to error signal samples of actual error event.

This paper introduces a new soft-reliability information-based post-Viterbi processor with improved noise-robustness. In new and conventional schemes, the procedures for finding a set of likely error starting positions and performing soft-reliability information-based error-correction over the set are the same. The main difference between the two schemes is how they obtain the soft-reliability estimate. The new scheme obtains more reliable soft-reliability information by removing the noise-sensitive component from the log-likelihood ratio of a posteriori probabilities. The new scheme can give minimum negative soft-reliability value at actual error starting position even when the channel noise is severe and consequently, the probability of miss-correction and no correction is significantly reduced. The performance has been evaluated for the magnetic recording channel. With only a few alterations, the technique can be applied to optical storage systems.

The paper is organized as follows. Section II introduces the new technique. In Section III, simulation results are given, and finally, conclusions are given in Section IV.

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II. A NEW POST-VITERBI PROCESSOR

This section firstly overviews conventional soft-reliability information-based post-Viterbi processor. Secondly, a new soft-reliability information-based post-Viterbi processor will be described.

A. Conventional soft–reliability information -based Post-Viterbi Processor

The performance of a partial response maximum likelihood (PRML) system can be improved by employing a soft-reliability information based post-Viterbi processor based on an error detection code that can correct a dominant error event at the output of the channel detector. In the soft-reliability information-based post-Viterbi processor based on error detection code [2], an error detection decoder computes a syndrome to check for the presence of errors in the estimated codeword, which is found at the output of the channel detector. When the syndrome is non-zero, the scheme is activated. Based on the syndrome, the scheme generates a set of all likely error starting positions for prescribed error events and then it attempts to find a position and its error-type with minimum one only if there exist positions that yield negative soft-reliability estimate over the set. Note that the number of position with negative soft-reliability value is mostly one, but rarely it is more than one because of severe channel impairments. If positions that produce negative soft-reliability estimate do not exist, the scheme does not make any correction. However, it is observed that the scheme makes no correction or miss-correction with high probability even when error occurs in the estimated codeword. As a result, the number of bit errors increases which can be a significant detrimental factor to the detection performance. We have observed that the problem happens when noise characteristic approximately matches to error signal samples of actual error event. For solving the problem, we propose a new soft-reliability information with better noise-robustness. The soft-reliability information of the conventional scheme is given as the log-likelihood ratio of the posteriori probabilities, but that of the new scheme is obtained by adding a correction component that reduces the sensitivity of channel noise to log-likelihood ratio of the posteriori probabilities. Then, the new information becomes more robust to noise compared to that of conventional scheme and consequently, the new scheme has a smaller bit error rate performance than the conventional one.

B. A New Soft-reliability Information-based Post-Viterbi Processor

We describe a sub-optimum post-Viterbi error correction scheme based on the new soft-reliability information. We assume that a bipolar codeword $\mathbf{b}=[b_0,...,b_{N-1}]$ of length *N*, generated by an error detection encoder, is transmitted over a partial response channel and corrupted by additive white Gaussian noise (AWGN).

The *k*-th input signal of the ML detector, r_k is expressed as

$$r_k=\sum_{i=0}^{l_h-1} h_i\cdot b_{k-i}+n_k=s_k+n_k\ , \qquad (1)$$

where b_k is a *k*-th bipolar coded bit of $\mathbf{b}$, h_k is a *k*-th element of a channel target response $\mathbf{h}_0^{l_h-1}=[h_0,...,h_{l_h-1}]$ of length l_h, n_k is an AWGN sample, and s_k is a *k*-th signal sample generated by the convolution of the bipolar coded sequence $\mathbf{b}$ and the channel target response $\mathbf{h}_0^{l_h-1}$. By definition, the ML detector selects a bipolar coded sequence $\hat{\mathbf{b}}=[\hat{b}_0,...,\hat{b}_{N-1}]$ that minimizes the Euclidean metric

$$\sum_{k=0}^{N-1}\left(r_k-\hat{s}_k\right)^2\ , \qquad (2)$$

where $\hat{s}_k$ is a *k*-th signal sample yielded the convolution of the output sequence of the ML detector $\hat{\mathbf{b}}$ and the channel target response $\mathbf{h}_0^{l_h-1}$, i.e.,

$$\hat{s}_k=\sum_{i=0}^{l_h-1} h_i\cdot\hat{b}_{k-i}\ . \qquad (3)$$

We try to design a decoding scheme that corrects one of prescribed error events occurred at the output of the ML detector that dominates the other error events. Let us assume that one of the prescribed error events $\left\{\mathbf{e}^{(i)},\ i\in\{1,\cdots,E\}\right\}$ occurs in a codeword, where *E* is the number of prescribed error events. Then, L_i likely error starting positions $\left\{p_j^{(i)},\ i\in\{1,\cdots,E\}\text{and } j=1,\cdots,L_i\right\}$ for $\left\{\mathbf{e}^{(i)},\ i\in\{1,\cdots,E\}\right\}$ from a syndrome computed by error detection code are given, where L_i is the number of likely error starting positions for a prescribed *i*-th error event. If the length of the error event $\mathbf{e}^{(i)}$ is $l^{(i)}$, and its error starting position is $p_m^{(i)}$ among $\left\{p_j^{(i)},\ j=1,\cdots,L_i \text{ and } 1\le m\le L_i\right\}$, then the error event $\mathbf{e}^{(i)}$ is given by

$$\begin{aligned}\mathbf{e}^{(i)}&=\left[e_0^{(i)},e_1^{(i)},\cdots,e_{l^{(i)}-1}^{(i)}\right]\\&=\left[b_{p_m^{(i)}},b_{p_m^{(i)}+1},\cdots,b_{p_m^{(i)}+l^{(i)}-1}\right]-\left[\hat{b}_{p_m^{(i)}},\hat{b}_{p_m^{(i)}+1},\cdots,\hat{b}_{p_m^{(i)}+l^{(i)}-1}\right]\\&=\mathbf{b}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}-\hat{\mathbf{b}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}\end{aligned} \qquad (4)$$

and the error signal vector, which is the convolution of the error event $\mathbf{e}^{(i)}$ and the channel target response $\mathbf{h}_0^{l_h-1}$, is expressed as

$$\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}=\left[\mathbf{b}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}-\hat{\mathbf{b}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}\right]*\mathbf{h}_0^{l_h-1}$$

$$= \mathbf{s}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} - \hat{\mathbf{s}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} . \quad (5)$$

Here, each error signal sample $s_k^{e^{(i)}}$ is

$$\begin{aligned} s_k^{e^{(i)}} &= \sum_{i=0}^{l_h-1} \left(b_{k-i} - \hat{b}_{k-i} \right) \cdot h_i \\ &= \sum_{i=0}^{l_h-1} e_{k-p_m^{(i)}-i}^{(i)} \cdot h_i \ \text{ for } \ p_m^{(i)} \le k \le p_m^{(i)} + l^{(i)} + l_h - 2 \end{aligned} , \quad (6)$$

where $e_j^{(i)} = 0$ if $j < 0$ or $j > l^{(i)} - 1$. Let $\hat{s}_k'$ be a k-th signal sample produced by the convolution of the flipped ML detector output sequence $\hat{\mathbf{b}}' = [\hat{b}_0', ..., \hat{b}_{N-1}'] = [\hat{\mathbf{b}}_0^{p_m^{(i)}-1}, -\hat{\mathbf{b}}_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}-1}, \hat{\mathbf{b}}_{p_m^{(i)}+l^{(i)}}^{N-1}]$, according to an error event $\mathbf{e}^{(i)}$ of length $l^{(i)}$ and corresponding likely error starting positions $p_m^{(i)}$ for $1 \le m \le L_i$, and the channel target response $\mathbf{h}_0^{l_h-1}$, i.e., $\hat{s}_k' = \sum_{i=0}^{l_h-1} \hat{b}_{k-i}' \cdot h_i$.

In [2], we derived a criterion of conventional soft-reliability information-based post-Viterbi processor that finds the most probable error starting position among all likely error starting positions $\left\{ p_j^{(i)},\ i = 1, ..., E \text{ and } j = 1, \cdots, L_i \right\}$ for the prescribed error events $\left\{ \mathbf{e}^{(i)},\ i = 1, \cdots, E \right\}$ is

$$\hat{\mathrm{P}} = \underset{\text{all } p_{j \in \{1,...,L_i\}}^{i \in \{1,...,E\}}}{\arg\min} \left(\underbrace{\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2} \left[\left(r_k - \hat{s}_k' \right)^2 - \left(r_k - \hat{s}_k \right)^2 \right]}_{\substack{\text{Soft-reliability information} \\ \text{(Log-likelihood ratio of posteriori probabilities)}}} < 0 \right) . \quad (7)$$

Equation (7) means that the error correction is performed based on an error starting position and its error-type associated with minimum one only when it exist likely error starting positions with negative soft-reliability value. For the most likely error starting position $\hat{\mathrm{P}} = p_m^{(i)}$ among $\left\{ p_j^{(i)},\ i = 1, ..., E \text{ and } j = 1, \cdots, L_i \right\}$ for $\left\{ \mathbf{e}^{(i)},\ i = 1, \cdots, E \right\}$, soft-reliability estimate in (7) is

$$\begin{aligned} &\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \left[\left(r_k - \hat{s}_k' \right)^2 - \left(r_k - \hat{s}_k \right)^2 \right] \\ &= -\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \left[\left(s_k^{e^{(i)}} \right)^2 + 2 \left(s_k^{e^{(i)}} \cdot n_k \right) \right] < 0 \end{aligned} . \quad (8)$$

For the other likely error starting positions $\left\{ p_j^{(i)},\ i = 1, ..., E \text{ and } j = 1, \cdots, L_i \,(j \ne m) \right\}$, soft-reliability value in (7) is

$$\begin{aligned} &\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2} \left[\left(r_k - \hat{s}_k' \right)^2 - \left(r_k - \hat{s}_k \right)^2 \right] \\ &= \sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2} \left[\left(s_k^{e^{(i)}} \right)^2 + 2 \left(s_k^{e^{(i)}} \cdot n_k \right) \right] > 0 \end{aligned} . \quad (9)$$

Actually, a conventional post-Viterbi scheme based on error correlation filter [1] is easily derived from the soft information estimate in (7). The error signal $e_k^{e^{(i)}}$ between the equalizer output r_k and the signal sample $\hat{s}_k$ is

$$e_k^{e^{(i)}} = s_k^{e^{(i)}} + n_k = \sum_{i=0}^{l_h-1} e_{k-p_m^{(i)}-i}^{(i)} \cdot h_i + n_k . \quad (10)$$

Then, the error signal is convolved with a bank of error correlation filters to find the most probable error starting positions and their error types of each prescribed error event. Let us assume that an error event $\mathbf{e}^{(i)}$ in estimated codeword occurs at $p_m^{(i)}$, which is one among $\left\{ p_j^{(i)},\ j = 1, \cdots, L_i \text{ and } 1 \le m \le L_i \right\}$. Then, for the error correlation filter corresponding to $\mathbf{e}^{(i)}$ with length $l^{(i)}$, the filter output $\mathbf{f}^{e^{(i)}}$ at $p_m^{(i)}$ becomes in vector notation

$$\begin{aligned} \mathbf{f}^{e^{(i)}} &= \left(\left[\mathbf{s}^{e^{(i)}} \right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \right)^{\mathbf{T}} \left[\mathbf{s}^{e^{(i)}} \right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \\ &\quad + \left(\left[\mathbf{s}^{e^{(i)}} \right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \right)^{\mathbf{T}} \cdot \left[\mathbf{n} \right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \end{aligned} , \quad (11)$$

where $[\mathbf{n}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}$ is the AWGN vector. Basically, equation (11) is supposed to give the maximum output compared to other error correlation filter outputs $\mathbf{f}^{e^{(j)}}$ corresponding to the most likely error starting positions of each prescribed error event $\mathbf{e}^{(j)}$, where $j = 1, ..., E$ and $j \ne i$. But, $\mathbf{f}^{e^{(i)}}$ often fails to give the maximum value, so that $\mathbf{f}^{e^{(j)}}$ for $j = 1, ..., E$ and $j \ne i$ gives the maximum and the miss-selection occurs. Moreover, although $\mathbf{f}^{e^{(i)}}$ yields the maximum compared to $\mathbf{f}^{e^{(j)}}$ for $j = 1, ..., E$ and $j \ne i$, it has been often observed that $\mathbf{f}^{e^{(i)}}$ can produce the incorrect error position (miss-positioning, $\left\{ p_j^{(i)},\ j = 1, \cdots, L_i \text{ and } j \ne m \right\}$). The main reason of the miss-correction is because of $\left(\left[\mathbf{s}^{e^{(i)}} \right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \right)^{\mathbf{T}} \cdot [\mathbf{n}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}$ in (11). We can roughly express the severe AWGN sequence, when the error event $\mathbf{e}^{(i)}$ occurs, as

$$[\mathbf{n}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \approx -\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} + [\boldsymbol{\varepsilon}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}, \quad (12)$$

where $[\boldsymbol{\varepsilon}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}$ is a vector that consists of any real numbers. When we plug (12) into (11), then $\mathbf{f}^{e^{(i)}}$ becomes

$$\begin{aligned}\mathbf{f}^{e^{(i)}} &= \left(\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\right)^{\mathbf{T}} \left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \\ &\quad -\left(\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\right)^{\mathbf{T}} \cdot \left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \\ &\quad +\left(\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\right)^{\mathbf{T}} [\boldsymbol{\varepsilon}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2} \\ &= \left(\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\right)^{\mathbf{T}} [\boldsymbol{\varepsilon}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\end{aligned}. \quad (13)$$

So, it is highly probable that the filter output $\mathbf{f}^{e^{(i)}}$ cannot yield the maximum output, so that it is prone to miss-correction. If this is the case, then we have to check soft-reliability estimates (8) and (9). Assuming severe AWGN sequence (12), we can rewrite the soft-reliability estimates (8) and (9). For the most probable error starting position $\hat{\mathrm{P}} = p_m^{(i)}$ among $\left\{p_j^{(i)},\ i=1,...,E \text{ and } j=1,\cdots,L_i\right\}$ for $\left\{\mathbf{e}^{(i)},\ i=1,\cdots,E\right\}$, the AWGN can be sometimes assumed as (12). Accordingly, equation (8) is

$$\begin{aligned}&\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(r_k-\hat{s}_k'\right)^2-\left(r_k-\hat{s}_k\right)^2\right] \\ &= -\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2+2\left(s_k^{e^{(i)}}\cdot n_k\right)\right] \\ &= -\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2-2\left(s_k^{e^{(i)}}\right)^2+2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)\right] \\ &= \sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2-2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)\right]\end{aligned}. \quad (14)$$

For the other likely error starting positions $\left\{p_j^{(i)},\ i=1,...,E \text{ and } j=1,\cdots,L_i\,(j\neq m)\right\}$, the AWGN can be assumed as $[\mathbf{n}]_{p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2} \approx [\boldsymbol{\varepsilon}]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}$ because there are no errors in the likely error positions. Accordingly, equation (9) is

$$\begin{aligned}&\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(r_k-\hat{s}_k'\right)^2-\left(r_k-\hat{s}_k\right)^2\right] \\ &= \sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2+2\left(s_k^{e^{(i)}}\cdot n_k\right)\right] \\ &= \sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2+2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)\right]\end{aligned}. \quad (15)$$

For non-error starting positions, the soft-reliability information (15) can produce positive value because of $\left(s_k^{e^{(i)}}\right)^2 > 2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)$ even if $2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)<0$. However, for the actual error starting position, the soft-reliability information (14) may fail to give negative value because $\left[\left(s_k^{e^{(i)}}\right)^2-2\left(s_k^{e^{(i)}}\cdot\varepsilon_k\right)\right]>0$. Note that the soft-reliability information for the actual error starting position usually gives negative value. Sometimes a positive value is yielded if the AWGN characteristic roughly equals $-\left[\mathbf{s}^{e^{(i)}}\right]_{p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}$. Clearly, compared to the other likely error starting positions, the soft-reliability information for the actual error starting position is a smaller positive value. Then, the threshold for correction of an actual error event may be not exactly zero, but slightly larger. Since determination of whether the estimated error starting position is correct depends upon the threshold zero, we need to have more reliable soft-reliability estimate.

For a more reliable soft-reliability estimate, we try to eliminate $-2\sum_k\left(s_k^{e^{(i)}}\cdot n_k\right)$ (noise-sensitive component) in (14) by adding $2\sum_k\left(s_k^{e^{(i)}}\cdot n_k\right)=2\sum_k\left[s_k^{e^{(i)}}\cdot(r_k-\hat{s}_k')\right]$ (correction component) to the conventional soft-reliability estimate, which is the log-likelihood ratio of posteriori probabilities. As a result, a new criterion for finding the most probable error starting position among $\left\{p_j^{(i)},\ i=1,...,E \text{ and } j=1,\cdots,L_i\right\}$ for $\left\{\mathbf{e}^{(i)},\ i=1,\cdots,E\right\}$ is

$$\hat{\mathrm{P}} = \underset{\text{all } p_{j\in\{1,\dots,L_i\}}^{i\in\{1,\dots,E\}}}{\arg\min}\left(\underbrace{\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\begin{array}{l}\left(r_k-\hat{s}_k'\right)^2-\left(r_k-\hat{s}_k\right)^2\\ +\underbrace{2(r_k-\hat{s}_k')\cdot s_k^{e^{(i)}}}_{\text{Correction term}}\end{array}\right]}_{\text{A new soft-reliability information}}<0\right). \quad (16)$$

For the most likely error starting position $\hat{\mathrm{P}} = p_m^{(i)}$ among $\left\{p_j^{(i)},\ i=1,...,E \text{ and } j=1,\cdots,L_i\right\}$ for $\left\{\mathbf{e}^{(i)},\ i=1,\cdots,E\right\}$, soft-reliability estimate in (16) is

$$\begin{aligned}&\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(r_k-\hat{s}'_k\right)^2-\left(r_k-\hat{s}_k\right)^2+2(r_k-\hat{s}'_k)\cdot s_k^{e^{(i)}}\right]\\&=\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[n_k^2-\left(s_k^{e^{(i)}}+n_k\right)^2+2\cdot n_k\cdot s_k^{e^{(i)}}\right]\\&=\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[n_k^2-n_k^2-2\cdot n_k\cdot s_k^{e^{(i)}}-\left(s_k^{e^{(i)}}\right)^2+2\cdot n_k\cdot s_k^{e^{(i)}}\right]\\&=-\sum_{k=p_m^{(i)}}^{p_m^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}\right)^2\right]<0\end{aligned}\quad. \tag{17}$$

For the other likely error starting positions $\left\{p_j^{(i)},\ i=1,\ldots,E \text{ and } j=1,\cdots,L_i\,(j\neq m)\right\}$, soft-reliability value in (16) is

$$\begin{aligned}&\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(r_k-\hat{s}'_k\right)^2-\left(r_k-\hat{s}_k\right)^2+2(r_k-\hat{s}'_k)\cdot s_k^{e^{(i)}}\right]\\&=\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\left(s_k^{e^{(i)}}+n_k\right)^2-n_k^2+2\left(s_k^{e^{(i)}}+n_k\right)\cdot s_k^{e^{(i)}}\right]\\&=\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[\begin{array}{l}\left(s_k^{e^{(i)}}\right)^2+2\cdot s_k^{e^{(i)}}\cdot n_k+n_k^2-n_k^2\\+2\left(s_k^{e^{(i)}}\right)^2+2\cdot s_k^{e^{(i)}}\cdot n_k\end{array}\right]\\&=\sum_{k=p_j^{(i)}}^{p_j^{(i)}+l^{(i)}+l_h-2}\left[3\left(s_k^{e^{(i)}}\right)^2+4\cdot s_k^{e^{(i)}}\cdot n_k\right]>0\end{aligned}\quad. \tag{18}$$

With the modified soft information, the soft-reliability information for an actual error starting position becomes equal to the square sum of the corresponding error signal. So, there is no ambiguity to decide the actual error starting position. In other words, the soft-reliability information (17), for the actual error starting position, becomes negative value irrespective of noise characteristic and the soft-reliability estimates (18), for the other non-error starting positions, are always positive value because of $3\left(s_k^{e^{(i)}}\right)^2 >> 4\cdot s_k^{e^{(i)}}\cdot n_k$. Therefore, the new soft-reliability information becomes more reliable and consequently, the probability of miss-correction and no correction of the new scheme is significantly reduced compared to that of conventional scheme.

III. SIMULATION RESULTS

A (203, 200) cyclic redundancy check (CRC) code generated by a generator polynomial $g(x) = 1+x^2+x^3$ is used as an error detection code [11] and the code can detect the dominant error events for perpendicular recording [9][10]. The bit error rates (BERs) of post-Viterbi processors are simulated and compared at user density D_u=1.4 with the channel target response of $\mathbf{h}_0^3=[1,6,7,2]$ ($\mathbf{h}(D)$ = 1 + 6D + 7D^2 + 2D^3 , where D is one-symbol delay) over perpendicular recording. The user density D_u is defined by $D_u = R \times D_c$, where R is the code rate and D_c is the channel density. The

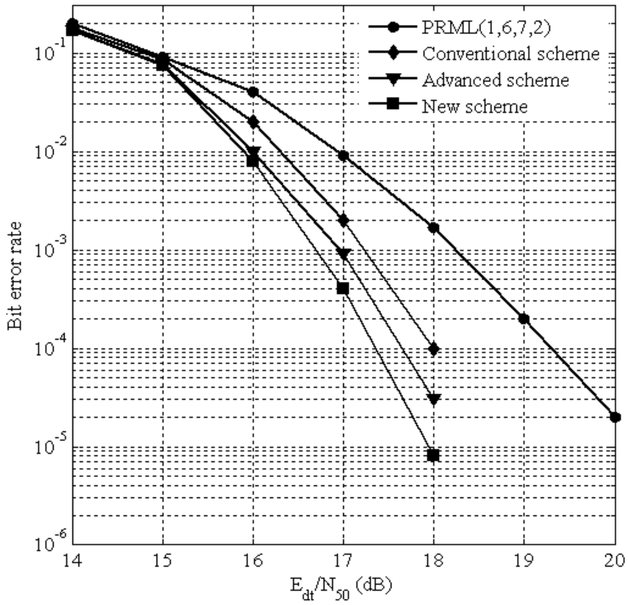

Fig. 1. Comparison of BERs of post-Viterbi processors under N_{50}.

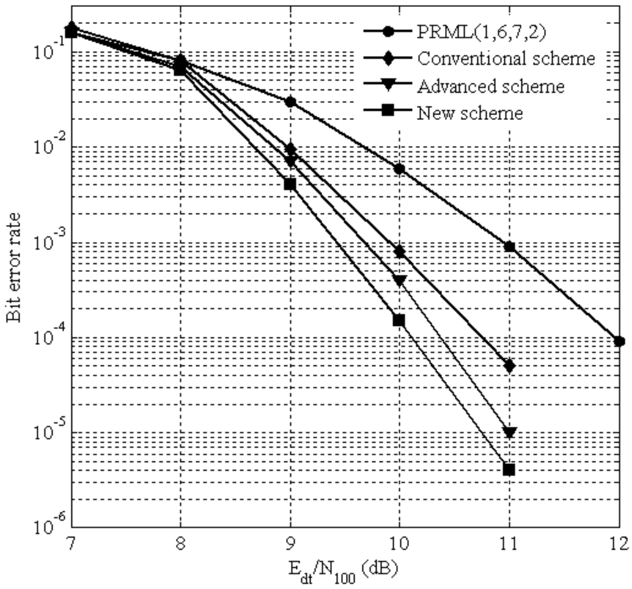

Fig. 2. Comparison of BERs of post-Viterbi processors under N_{100}.

dominant error events (E=6) at user density 1.4 are listed in [1][9][10]. As a reference, the BER of a PRML (1, 6, 7, 2) system is also shown at the same user density 1.4. For the user density 1.4, the corresponding channel densities for the PRML (1, 6, 7, 2) system and post-Viterbi processors are 1.4 (=D_u /R=1.4/1) and 1.42 (≈D_u/R =1.4/(200/203)), respectively. The signal-to-noise ratio (SNR) has been defined in [10] as the ratio of the energy of the first derivative of the transition response E_{dt} and the noise spectral density. In our simulations, the noise parameter N_{50} or N_{100} in the SNR definition signifies a mixture of 50% AWGN and 50% jitter noise or 100% AWGN, respectively. Fig. 1 shows the BERs of conventional, advanced and new post-Viterbi processors under N_{50}. In Fig. 1, the legend “conventional scheme” [1] corresponds to an error correlation filter-based post-Viterbi processor that determines the most probable error starting position and its error-type based on a likelihood value, “advanced scheme” [2] means conventional soft-reliability information-based post-Viterbi processor that chooses the most likely error starting position based on soft-reliability estimate, and “new scheme”, which is proposed in the paper, is a new soft-reliability information-based post-Viterbi processor that supplies more reliable soft-reliability estimate than “advanced scheme. All post-Viterbi processors produce considerable performance gains compared to conventional PRML (1, 6, 7, 2) system. Among the post-Viterbi processors, the “conventional scheme” results in worst performance because the scheme does not have any criterion for judging whether an estimated error starting position is correct. Thus, the scheme accomplishes error correction based on an error starting position and its error-type corresponding to the maximum likelihood value whenever the syndrome of a codeword is non-zero. In the “advanced scheme”, the miss-correction of the dominant error events is reduced compared to “conventional scheme” because the scheme attempts to perform error correction only if there exist error starting positions whose the soft-reliability estimates are negative. The “new scheme” yields the best performance among the schemes considered here. As shown in Fig. 1, while “advanced scheme” suffers from no correction and miss-correction due to channel noise, the newly proposed soft-reliability information-based scheme overcomes the problem by using a more reliable soft-reliability estimate. Fig. 2 shows a comparison of the BERs of conventional, advanced, and new post-Viterbi processors under N_{100} . The result shows the same performance trend as shown in Fig. 1, irrespective of the noise distribution.

IV. Conclusion

We have investigated a new soft-reliability information-based post-Viterbi processor that reduces the probability of miss-correction and no correction of conventional scheme. We have found that miss-correction and no correction of the conventional scheme result from the soft-reliability information with noise-sensitive component, which is given as the log-likelihood ratio of posteriori probabilities. We have proposed more reliable soft-reliability information with better noise-robustness obtained by adding correction component to the log-likelihood ratio of posteriori probabilities. By simulation, we have found that probability of miss-correction and no correction of the new scheme is considerably reduced compared to that of conventional scheme. As a result, we conclude that the new technique is a good candidate for high-capacity storage systems.

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