Guided Scrambling: A New Coding Technique for Holographic Storage

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1 Introduction

Holographic storage of digital data is a promising technique for reliably storing mass data [1]. In holographic storage entire two-dimensional (2D) pages of data are intermingled with each other such that numerous pages of data are recorded in a common volume of the storage medium. Certain pages with vexatious data patterns should be avoided as they could easily interact with other pages. In particular data with a periodic pattern give rise to distinct peaks of intensity in the Fourier transform plane. Vardy et al. [2] stipulated that the data patterns should have as many as transitions from light to dark and vice versa as possible in each column and row of a page. This may be achieved by requiring that in each row and column, there are at least $t$ ($t$ is a positive prescribed integer) transitions of the type "1" $\rightarrow$ "0" or "0" $\rightarrow$ "1". Such an array is called a conservative array of strength $t$. There is another requirement for holographic storage that the number of "1"s and "0"s in a two-dimensional array should be balanced, so that the content of the data pages will be independent of the amount of signal light for recording.

Guided scrambling [3] is a member of a larger class of related coding schemes called multi-mode code. In multi-mode codes, each source word can be represented by a member of a selection set consisting of $L$ codewords. The encoder evaluates the "quality" of each codeword in the selection set, and transmits the codeword that "best" matches the quality criterion at hand. The members of the selection sets are not judiciously chosen and stored in memory, and are randomly picked. The basic idea is that, provided the selection set is sufficiently large, we will find, with high probability, an adequate codeword fulfilling the constraints. The advantage of this approach is clear: at the encoder site we only need a) a simple mechanism for translating source words into "random" codewords and b) a mechanism for evaluating the "quality" of the candidate words. The stumble block of conventional large codes and the look-up table can be avoided, and replaced by a simple randomization algorithm.

2 Description of the Coding Scheme

For a data page of holographic storage, assume a source word of length $n$ bits is given, the source word is preceded by $p$ redundancy bits. In the final coding stage the $n + p$ bits are arranged in a matrix (page). We produce the set $B_p$, which comprises all $n + p$-bit words found by augmenting the source word with all possible $2^p$ redundant $p$-bit words. Each $(n + p)$-bit word is forwarded to a scrambler, and the output of the scrambler is arranged in the required matrix form. The scrambling sequence can be obtained through the following feedback polynomial:

$$s_k = b_k + g_1s_{k-1} + \cdots + g_n s_{k-n},$$

(1)
where $s_k$ denotes the scrambled data, $b_k$ the input data, and $g_n$ the coefficients of the scrambler. Both the input data, output data, and coefficients $\in \{0, 1\}$, the multiplication and the addition are modulo-2. The largest index value of the non-zero coefficient is referred to as the order of scrambler. With a certain structure, the scrambling output sequence has a long periodicity and small correlation, and it is thus called pseudo-random sequence. After processing all possible arrays, a matrix taken from the set of $2^p$ generated matrices that best fulfills the two requirements for holographic storage will be chosen and stored in the holographic medium. In Section 3, we will show that such kind of sequence can be found with quite large probability.

Note that there is no necessity to transmit additional information to the receiver in order to decode the original data. The decoder just uses the same method to descramble the received data sequence with the following relation:

$$b_k = s_k + g_1 s_{k-1} + \cdots + g_n s_{k-n}.$$  

Clearly, such a kind of coding scheme has the advantages of reliability and simplicity. Error propagation is limited to the number of non-zero coefficient of descrambler, which is not a big deal. The number of preceding bits, $p$, is considered as redundancy. As $p$ is relatively small, it offers high coding efficiency.

3 Results

We design the encoder for the two-dimensional $16 \times 16$ array. The scrambler order is 11, and the configuration of the scrambler is

$$s_k = b_k + s_{k-2} + s_{k-11}.$$  

Note that the method cannot guarantee to find an array with specified constraints, though if the intermediate set is sufficiently large such an array is found with probability approaching unity. For example, Table 1 lists the probability $Pr$ that a balanced array (an array with equal numbers of 'zero's and 'one's) is not found in a population of size $2^p$ of random $16 \times 16$ arrays.

<table>
<thead>
<tr>
<th>p</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.80E-002</td>
</tr>
<tr>
<td>7</td>
<td>1.44E-003</td>
</tr>
<tr>
<td>8</td>
<td>2.08E-006</td>
</tr>
<tr>
<td>9</td>
<td>4.33E-012</td>
</tr>
<tr>
<td>10</td>
<td>1.88E-023</td>
</tr>
<tr>
<td>11</td>
<td>3.53E-046</td>
</tr>
</tbody>
</table>

Table 1: Probability $Pr$ that a balanced $16 \times 16$ array is not found in a set of size $2^p$.

In the simulation we first choose the balanced (equal numbers of 1's and 0's) array, then investigate the transition property of each row and column. As is disclosed in Table 1, the larger the value of $p$, the more balanced the arrays can be found. Thus there is more choice to find a better conservative array. Different values of $p$ affect the “randomization” of the recorded data page, that is the strength of the conservative array. We investigate this characteristic by collecting the distribution of the strength $t$ for different values of $p$. Table 2 shows the result. It is shown that as $p$ is increased, the strength $t$ will approach larger values with little variance. For example, when $p = 11$, all arrays have strength $> 4$. 

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Table 2: Distribution of strength \( t \) in 2-D conservative array

<table>
<thead>
<tr>
<th>( p )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.0003</td>
<td>0.1364</td>
<td>0.7864</td>
<td>0.0768</td>
<td>0.0001</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>0.0170</td>
<td>0.8411</td>
<td>0.1415</td>
<td>0.0004</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.7254</td>
<td>0.2734</td>
<td>0.0007</td>
</tr>
<tr>
<td>11</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.5254</td>
<td>0.4736</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

We finally give an example in Fig. 1 to illustrate the guided scrambling. The input is a special pattern, the number of preceding bits is 10, the size of the array is 16 \( \times \) 16, thus the code rate is 0.961. Note that in Fig. 1 the strength of the output array equals 5.

![Input data array](a) ![Output of the Guided Scrambling encoder](b)

Figure 1: (a) Input data array; (b) Output of the Guided Scrambling encoder

4 Conclusions

We have investigated the properties of guided scrambling in the context of holographic storage. We have found that guided scrambling offers, with a small cost in terms of code redundancy, an effective tool for removing detrimental pages.

References

